Mathematics 114 Information Sheet 2019

Essential Information

Lecturers

Group 1.1 (Afr) *Lecturer & Course Convenor:* Dr Karin-Therese Howell (kthowell@sun.ac.za)

Office 1009D, Mathematical Sciences & Industrial Psychology building

Lectures: Natural Sciences Building room 2020: Mo 3, Tu 1, Thu 2 & 4, Fri 4

Group 1.2 (Eng) *Lecturer:* Dr James Gray (jamesgray@sun.ac.za)

Office 1016A, Mathematical Sciences & Industrial Psychology building

Lectures: RW Wilcocks room 3001: Mo 3, Tu 1, Thu 2 & 4, Fri 4

Group 2 (Eng) *Lecturer*: Prof Ingrid Rewitzky (rewitzky@sun.ac.za)

Office 1022A, Mathematical Sciences & Industrial Psychology building

Lectures: Natural Sciences Building room 3004: Mo 4, We 3, Thu 2, Fri 1 & 3

Language Policy

All course notes will be made available in both English and Afrikaans. Please note; however, that each lecturer may use their own discretion about posting class notes on SunLearn.

Information & Communication

Module information, tutorials, online tests, memoranda, marks and auxiliary material will be posted on the SunLearn page at http://learn.sun.ac.za. *This is the official means of communication for the module*.

You should have automatic access to the module's web page once you have registered, report any problems immediately at NARGA (Natural Sciences Computer Users' Area).

There will be weekly compulsory online tests on SunLearn based on the work of the previous week.

Textbook

You need to have a copy of the *prescribed textbook*: CALCULUS 8th Edition, James Stewart.

Books are available at Van Schaiks and Protea bookshops. It should also be possible to find second hand copies of the 6th or 7th Editions, but the onus rests on you to check any discrepancies. The purchase of the "Student Solution Manual" which comes with this textbook is optional.

Tutorials

Tutorials are on Wednesdays and Thursdays from 14:00 to 16:00. You will be assigned to ONE of these afternoons.

This course makes use of a Differentiated Tutorial structure. Initially students will be divided into Group 1 (Basic skills) or 2 (Problem solving and proofs) based on their NBT Mathematics scores. Repeaters and EDP students will be placed in Group 1. After the EA test, high risk students will be identified and moved to Group 1.

The tutorials will be used to work on a selection of problems in the form of a SmartTut which is released every Friday on SunLearn. It will be on the work of the previous week. It is each student's own responsibility to complete these problems to ensure that they understand the material. The first hour of each tutorial will be dedicated to an apprentice-style tutorial, the next to individual work. There will be a weekly vote which closes on Fridays, where students vote for the problems they need additional assistance on. Some of these will be selected for discussion in the first hour of the following week's tutorial. Every Monday a video on an example of the week's work will be released on SunLearn to assist students.

There will also be three project tutorials where you will use the time to work on a project in a group. These are indicated on the Course Schedule.

ATTENDANCE OF LECTURES AND TUTORIALS IS COMPULSORY, even for repeaters. According to university regulations you are not allowed to register for two modules at the same time if there are timetable (class and test) clashes. In special cases permission can be obtained for incomplete attendance of classes. To apply for this you must fill in a form at the office of the Mathematics Secretary (Mrs Adams), Office 1008C, Mathematical Sciences & Industrial Psychology Building by Friday, 8 February. *However; there will be no alternate arrangements made for tests.*

Tutorial attendance is recorded in a register. Permission to be absent from a tutorial can only be obtained with a valid medical certificate or written permission from the Registrar. These must be handed in to the secretary, Mrs Adams (room 1008C in the Maths building).

Assessments

- This module makes uses of Flexible Assessment.
- The following assessments are compulsory:
  - **Online Tests** available on a weekly basis. The average of these tests will form your **Online Test mark (OT).**
  - The **Early Assessment test (ET)** written on the 4th March 2019.
  - **Assessment Opportunity 1 (A1)** written on the 15th April 2019.
  - **Assessment Opportunity 2 (A2)** written on the 29th May 2019.

- The **Final Mark (FM)** will be determined as follows:

  FM = (0,05 x OT + 0,15 x ET+ 0,3 x A1 + 0,5 x A2) / 100. (1)
- If you miss the Early Assessment test as a result of illness or a valid reason, your final mark will be determined as follows:

  FM = (0,05 x OT + 0,45 x A1 + 0,5 x A2) / 100. (2)

- A third **Assessment Opportunity (A3)** written on the 20th June 2019.
- A3 is offered to all students who wrote A1 and A2 or missed either A1 or A2 (but not both) as a result of illness or a valid reason.
- If you write A3 as a result of illness or a valid reason, it will replace the assessment you missed in the calculation of (1) or (2).
- In the event that you write A1, A2 and A3 your final mark will be the highest of the following marks:
  - FM as in (1) or (2)
  - FM as in (1) or (2) with A3 replacing A1
  - FM as in (1) or (2) with A3 replacing A2.

Very Important

- Students have to obtain at least 50% for their final mark to pass.

- If you miss an assessment due to illness you need to hand in a valid medical certificate, otherwise you will be given 0 for the test.
- By the university rules a medical certificate must be issued within 24 hours from the time of assessment and handed in to Mrs Adams (room 1008C in the Mathematics building) within 7 days of the assessment date.
- Please refer to the university yearbook for the rules with regard to examinations and absence from classes and/or tests and requirements for medical certificates.
- Please note that lecturers may follow up medical certificates and verify that they are authentic.
- Any other reason for missing an assessment, has to be approved by the course convenor.
- Calculators are not allowed in tests or exams.

Module outcomes

After successfully completing Mathematics 114, the student should

- Have a good understanding of limits
- Have a good understanding of derivatives
- Be proficient in calculating derivatives symbolically
- Be proficient in using derivatives to solve optimization problems
- Have a basic understanding of integration using Riemann sums
- Understand the relation between differentiation and integration
- Have developed problem solving skills in the form of:
    - Designing proofs to simple statements
    - Approaching, analysing and solving simple unseen problems
- Have developed critical thinking in the form of:
    - Analysing proofs and identifying simple fallacies
    - Constructing counterexamples to false statements
- Have improved their habits of studying mathematics

From the module outcomes, you may realize that there are a few components that you may not have encountered at school. These consist of **proof writing**, **problem solving** and **counterexamples**. The reasons for these components is given in the following paragraphs.

Proof writing

Writing and understanding proofs is very important in mathematics and students taking Mathematics 2 already need some competence in writing proofs. This is a new concept to most students and therefore many struggle when they encounter proofs at the second year level. The goal of the proof writing component of the course is to give an introduction to proof writing so that they may better cope with the requirements of the second year.

There are two parts to the proof writing component. On the one hand, students will learn to write their own proofs to simple statements. On the other hand, understanding proofs from the textbook is also grouped under this heading. We will not ask you to write down a proof from memory. Rather, we will give you a proof from the textbook, possibly with some omissions, and ask you to explain certain steps or fill in the missing details. There will be examples of both kinds in the tutorials to prepare you for the assessment opportunities.

Problem solving

Problem solving is what you do when you don't know what to do. The kind of problems we ask in this section are not meant to be extremely difficult, but rather to see if you can apply your knowledge in a context different from the ones that you "trained" during tutorials. For example:

Instead of asking "What is the derivative of x^2+1 at the point (3,10)?" we may ask "Find the point on the curve y=x^2+1 where the tangent line has gradient 10."

Thus, the changes will be subtle, but to be able to do the questions you will need to understand what you are doing and not rely on memorizing methods.

Counterexamples

There are a number of reasons to ask students to construct counterexamples to certain statements. There is no algorithm or procedure that will allow you to solve any counterexample problem and therefore encourages students to understand the material rather than practice a certain collection of methods. Students also come to university with many misconceptions, and develop new misconceptions during their studies (this also applies to professional mathematicians when they were students!). Having a list of counterexamples and thinking about why their corresponding statements are false gives a way of correcting these misconceptions and therefore strengthens the students' mathematical understanding.

Example: Provide a counterexample to the false statement:

For all real numbers x and y, sin(x+y)=sin(x)+sin(y).

Projects

There will be three projects to work on during the semester. Three tutorials will be replaced with a session in which students can work on the project, discuss it with their group and ask assistants to help them. The projects will involve putting together different pieces of mathematics, including material that you learn throughout the semester as well as material that you learned at school. The reason for this is that you get a better understanding of material if you see how it interconnects with other material rather than a collection of unrelated techniques. The assistants will be instructed to help you think about the problem. They will not provide answers. The reason for this is that coming to grips with the project take time and thought. Getting the answer may give you the impression that you understand the project when you don't.

Assessments

The assessments (tests and exams) will be designed to take the module outcomes into consideration. The components alluded to will carry the following weightings in each of the assessments:

- **Basic skills at least 50%.** This includes the type of problems where we ask to perform a certain technique that we showed you in class and you practiced during the semester.
- **Advanced skills at most 50%.** This will include more conceptual problems, including problem solving, proof writing and counterexamples.

Each assessment opportunity will also have a 20% component that is about one of the projects. The questions in this section will be divided in the same percentages as outlined above.

Module outline

Self-study: Revision of coordinate geometry & straight lines
These are topics that you should be familiar with from school. Some students forget some of this material by the time they start their first year, and therefore need to revise them. These topics are all required for Calculus topics later on.

Week 1: Sets and logic
Everything in mathematics builds on sets and logic.

Week 2 & 3: Numbers, inequalities, absolute values, trigonometry and radian measure, functions, inverse functions
We spend some time revising numbers, how to solve basic inequalities and trigonometry. We introduce radian measure, the absolute value function and solve some absolute value inequalities. Most students will have seen functions at high school, but for Calculus you will need an improved understanding of functions. We will discuss properties of functions that you may not have come across, relate them to sets and statements, and later revisit them from the point of view of Calculus. We introduce the concept of the inverse of a function and what it means to be injective (one-to-one).

Week 4: Limits
The concept of a limit is one of the most important concepts in the course and underlies two of the other important concepts, derivatives and definite integrals.

Week 5: Continuity & Derivatives, Basic Properties and Functions
We study continuity and introduce the derivative. It is the most important concept in this course, and most of the course revolves around it. First we define it and develop some properties, then we learn how to differentiate most elementary functions, and then we show how it can be applied.

Week 6: Induction & the Binomial Theorem
Induction is a proof technique that entails finding a pattern and explaining why it always continues. This is a very useful technique for finding and proving formulas for various things. We also study the Binomial Theorem and applications of it.

Week 7: Derivatives. Trigonometric Derivatives and Product/Quotient/Chain Laws
Here we develop the techniques that allow us to differentiate most elementary functions.

Week 8: Exponential & logarithmic functions & Related Rates
We show how to make sense of arbitrary exponential functions and define logarithmic functions as their inverses. We discuss an application of derivatives.

Week 9: Rates of Change, Max & Min values
The first section gives you some idea of how Calculus may be used in practice. We also begin develop the theory that allow us to use Calculus to find minimum and maximum values. This discussion continues into the next week.

Week 10: Mean Value Theorem, How derivatives affect the shape of a graph
Towards the end of the week we begin studying how derivatives affect the shape of a graph, the start of learning how to sketch and read a graph.

Week 11: Infinite limits, Horizontal asymptotes and Graph sketching
The reason for learning how to sketch graphs of functions is to consolidate the understanding of the relation between a function and its derivative and to be able to read a graph. As exercises you will need to sketch the graphs of some functions to see this principle in action.

Week 12: Anti-derivatives and Definite integrals
If you want to know if you understand something, see if you can do it in reverse. But that's not the only reason for introducing anti-derivatives. They will also provide a connection to integration. Definite integrals are a way of calculating areas.

Week 13: Fundamental Theorem of Calculus
The Fundamental Theorem provides a connection between differentiation and integration and is the most important result in Calculus, since it allows the efficient calculation of integrals.

Week 14: Optimisation, Newton's Method
As said earlier, optimisation is probably the main application of differentiation and is how it will mostly be applied in practice.

One of the outcomes of the course is that students can form basic mathematical arguments. To achieve this, we spend some time on how to write proofs throughout the course. This skill will be further developed throughout the semester.

Mathematics Study Guide

University mathematics is different from high school mathematics. The material is a little more difficult, but the main stumbling block is that the pace is much faster. This section is designed to help you plan your studies in order to be successful in your Mathematics modules. One of the outcomes of this module is to have improved your study habits to help you in the modules to follow. This doesn't happen by simply reading this guide. You need to refer back to it once a week to see if there is any way you can improve.

1. **Take responsibility.**
   There is no-one to check up on you in university. You need to develop the motivation and self-discipline needed to study regularly throughout the semester and keep up with the material. The lecturers are there to aid you in your studies, but ultimately you are the one responsible for them.

2. **Don't fall behind.**
   The pace is much quicker and getting even one week behind will mean struggling for the duration of the course. There is no time for lecturers to give dozens of examples of a certain type of problem; often three to five are all they have time for. It is then up to the student to do more examples of the same type, in order to improve understanding and to test whether they understood. Preferably, these exercises have to be done soon after the lecture, ideally on the same day.

3. **Set up a work schedule.**
   You will have many subjects, and you will need to balance your time and energy between all of them. At some times during the semester one or more subjects will be more demanding than the others, due to projects or tests. But don't fall behind in Mathematics during these times. You can halve the time you would spend, and catch it up in the following week, but don't neglect it completely.

4. **Sixteen Credits means 160 hours.**
   The 16 credits for Mathematics 114 means that the average student should spend 160 hours on it during the semester. About 100 of these hours are the lectures and tutorials, so the average student should spend about 60 hours by themselves during the semester, or approximately 4 hours per week. In some subjects you can catch this up before the exam, but in Mathematics, these hours are better spent during the semester.

5. **Don't aim for 50%.**
   There is a famous joke that if you get 51% you neglected your friends. In mathematics, getting 50% usually spells disaster for your subsequent mathematics modules. If you understand Mathematics 114 only 50%, then how are you going to cope with the more demanding material of Mathematics 144 or the second year? If you aim to get 100%, you will find the subsequent module more manageable even if you can't achieve this grade.

6. **Read your textbook.**
   The textbook often has a different explanation of a concept that will strengthen your understanding. The examples are also often different, which will help you by giving you a larger base of examples. People have different preferred methods of learning. Some learn better when seeing someone else doing something, and some learn better when reading. The easy way out is

to claim that you learn better from lectures, but remember that you must take responsibility! Using both in combination is better than using only one.

7. Read it twice or more.

The first time, just skim through to see what is covered. Then read the examples and try to understand every step. Then read the theoretical development, even if that came before the examples. Try to understand the theory in relation to the examples. If needed you can read it once more to summarize the most important concepts and to fit them with the other concepts that you have learned so far in the course.

8. Fill in the details.

Sometimes, a textbook can skim over certain details. A problem in section 3 might start off needing concepts from section 3, but to finish it, you will need concepts from section 2. The book might then give the details of the concepts related to section 3, but skim over the details of the concepts of section 2. This is an excellent opportunity to revise and to improve your skills by treating it as an exercise to fill in the details of how each line follows from the next.

9. Skip it and come back to it.

If there is something you can't figure out (say in 30 minutes), don't be afraid to skip it. Try to find a way of doing the exercises nonetheless. Doing them will improve your understanding and prepare you to come back to the material later. But you must make a note of what you didn't understand – in the textbook and in a workbook – so that you can revisit it the following week.

10. Do exercises from the textbook.

The exercises assigned in tutorials will not be enough for many students. If you are unsure about how to do a certain type of problem, the best way is to do more of them until you feel comfortable.

11. Don't read the solution too soon.

Students often lull themselves into a false sense of security when reading solutions. The solution makes sense so they think they know what's going on. But when a problem is given without a solution, they don't know what to do. So, rather spend 10 or 20 extra minutes trying to solve the problem. If you still can't do it, start reading the solution, but keep all of it covered, and uncover it line by line. If you read a line that you didn't think of in your solution, stop, and try to finish the solution from there. If you can't do this after 5 minutes, then read the next line, etc. This takes more time and energy than simply reading a solution, but is immensely more effective in developing your understanding, skills, confidence and retention of the material.

12. Do fewer problems, but spend more time and energy on them.

Many students finish a problem and then stop and immediately move onto the next problem. This wastes a valuable learning opportunity. A lot can be gained from analysing the problem and solution.

Doing all the exercises at the end of each section might not be the best way to spend your time, especially when you have worked out how to do them. At that point you are simply repeating things that you already know how to do. While there is still value in that, a more productive way to spend your time is to analyse how you solved the problem and what was required. Try the following:

- Which concepts were involved in the problem, and which were the most important?

- How would the solution change if the given information was changed?
- Which changes to the given information would make your solution invalid?

13. Write out proper solutions.
When you have finished the computations to a problem, write out the solution as if you were explaining it to a friend who had missed the class on that topic. This will solidify your understanding, it will be useful if a friend or lecturer has questions, and when reviewing the material for the exam. If this seems too much, try to do this for at least one in every four problems.

14. Discuss problems with friends.
One of the best ways to learn mathematics is to discuss it with your peers. Your friend is often in a better position to explain something than the lecturer, since they recently figured it out for the first time, and that is what you need to do. It is also an opportunity to discover common mistakes and to talk about why they are mistakes. But remember to take responsibility and write you're your own solutions at the end. For such discussions it is useful to have clearly written solutions so that you can analyse your shortcomings and differences in solutions.

15. Don't be afraid to sound stupid.
Remember, you don't know the material at the start, but you want to learn throughout the semester and know it at the end. To do this, you need to analyse what have trouble with and to spend time on problems that improve this aspect. In order to analyse this, you often need to show your solutions to others, try to explain it to others in your own words, so that they may help you if you are wrong.

This is a very difficult skill to learn, since no-one wants to be wrong or sound stupid. But it is the most important process in identifying where you need to improve. If you keep in mind that this is done with the goal of improving and getting a better mark, it is actually the smart thing to do!

16. Take notes in class.
If you intend to spend 50 minutes just listening, you will probably lose interest in the first 20 minutes. Taking notes keeps you involved in the lecture. This way you also have the material to revise before the test and exam.

17. Don't copy lectures word for word.
Copying word for word is very passive, because it is possible with minimal concentration. Trying to rephrase things, or to identify the main point forces you to concentrate in class and therefore helps your understanding more.

18. Have a plan for your notes and your exercises.
You will want to revise notes before the test and exam, and will be a great benefit if your notes are organised at those times. For this to happen it is useful to start off with a plan. For example:

Have two books – one for taking notes in class and one for the exercises and tutorials you do throughout the semester. Take your class notes book to every lecture, start each lecture on a new page, number the lecture (e.g. by week + day), leave some space on the right half so that you can add comments when revisiting. In the exercise book, always indicate clearly which section you are working on and the number of the exercise. Start doing your rough work, but once you have finished figuring it out, indicate clearly where your neat solution starts and write it out clearly. It may be useful to use different colours for your rough work and neat solutions (or a nicer pen for the neat solutions). You will see the benefit of neat solutions when revising your work.

19. See your lecturer immediately.

When there is something you don't understand after some effort, make an appointment to see your lecturer immediately. It will be exponentially better to talk about the concept immediately than later. Other concepts will build on it, and if you don't see your lecturer immediately, there will be more and more concepts that you don't understand. Your lecturer will appreciate the fact that you are trying to keep up to date, rather than leaving everything for before the test.

20. Be prepared for consultations.

In order to use your consultation time efficiently come prepared. Mark clearly which exercises or concepts you need help with, and try the exercises first, so that the lecturer can help you at the point where you get stuck.

21. Ask specific questions.

For example: "How is a square root different from a logarithm?" instead of "Can you explain logarithms?" or "In this exercise, I did the following, but then got stuck." Instead of "Can you explain this method?" (although the latter is still OK). This will concentrate your and your lecturer's time on the most important parts, and it shows your lecturer that you have put in some effort on your own before seeing them.

22. Find your own study method.

The guidelines mentioned above are based on the lecturers' own experience, but the way you study may be different from this. Experiment and try to find what works for you, but do take the suggestions seriously. Studying university mathematics is more intensive than high school mathematics, and most students will need to adapt their study habits to cope.