Exchange programme Vrije Universiteit Amsterdam

Vrije Universiteit Amsterdam - Exchange programme Vrije Universiteit Amsterdam - 2024-2025

Exchange

Vrije Universiteit Amsterdam offers many English-taught courses in a variety of subjects, ranging from arts & culture and social sciences, neurosciences and computer science, to economics and business administration.

The International Office is responsible for course approval and course registration for exchange students. For details about course registration, requirements, credits, semesters and so on, please visit the exchange programmes webpages.

Measure Theory

Course Code	X_401028
Credits	6
Period	P1+2
Course Level	300
Language Of Tuition	English
Faculty	Faculty of Science
Course Coordinator	dr. W. Kager
Examiner	dr. W. Kager
Teaching Staff	dr. W. Kager
Teaching method(s)	Lecture, Written partial exam, Seminar

Course Objective

After this course...

1. ... the student will know and understand the basic concepts of measure theory and the theory of Lebesgue integration.

2. ... the student will understand the main proof techniques in the field, and he will also be able to apply the theory abstractly and concretely.

3. ... the student will be able to write elementary proofs himself, as well as more advanced proofs under guidance.

4. ... the student is able to use measure theory and integration in Riemann integration and calculus.

5. ... the student is able to work with Lebesgue measure and to exploit its special properties.

Course Content

We motivate and introduce the notion of a measure, that is, a way to assign a size to as many subsets as possible in an abstract space. It turns out that it is in general not possible to measure all sets, at least if one insists on countable additivity of the measure. This leads to the notion of a sigma-algebra. We show how one can obtain a unique measure on a sigma algebra once certain basic properties are imposed.

Once we have defined measure, we can introduce and discuss so called measurable functions which, roughly speaking, form the class of functions which we will be able to integrate. We then introduce and study integration of these measurable functions with respect to a measure. We discuss (among other things) the monotone and dominated convergence theorems concerning the interchangeability of limit and integral, the substitution rule, absolute continuity and the relation of this new integral to the Riemann integral. We also discuss multi-dimensional Lebesgue measures, product measures, and Fubini's theorem.

The theory leads to a new perspective on integration of functions, which is not only more general than the Riemann setting when working on the real line, but also allows one to integrate in an abstract setting. This is of crucial importance for the development of functional analysis and probability theory.

Additional Information Teaching Methods

Lectures and exercise classes.

Method of Assessment

Written final exam, and a written midterm exam after 7 weeks. The final

exam will be 50% of the final grade, and the midterm exam will be 50% as well.

Literature

Rene L. Schilling: Measures, Integrals and Martingales, Cambridge
University Press.

Additional Information Target Audience

3MAT, 3EOR

Recommended background knowledge

Single Variable Calculus, Multivariable Calculus and Mathematical

Analysis (or equivalent).