Exchange programme Vrije Universiteit Amsterdam

Vrije Universiteit Amsterdam - Exchange programme Vrije Universiteit Amsterdam - 2024-2025

Exchange

Vrije Universiteit Amsterdam offers many English-taught courses in a variety of subjects, ranging from arts & culture and social sciences, neurosciences and computer science, to economics and business administration.

The International Office is responsible for course approval and course registration for exchange students. For details about course registration, requirements, credits, semesters and so on, please visit the exchange programmes webpages.

Mathematical Analysis

Course Code	XB_0009
Credits	6
Period	P4+5
Course Level	100
Language Of Tuition	English
Faculty	Faculty of Science
Course Coordinator	G. Benedetti
Examiner	G. Benedetti
Teaching Staff	G. Benedetti
Teaching method(s)	Written partial exam, Seminar, Lecture

Course Objective

After this course, the student can

- put a rigorous foundation to the completeness of real numbers via the notion of supremum of a set;
- phrase the concept of sequential and functional limits in the challenge-response framework and use it to prove the fundamental convergence theorems of **R** and to compute concrete limits;
- apply the theory of convergence to series of real numbers and to the Banach contraction theorem on the real line;
- determine if a subset of R is closed or open and construct new open and closed sets starting from known ones;
- classify real functions according to their regularity (continuous, differentiable, Riemann-integrable), and employ their fundamental theorems in examples;
- compute the pointwise and uniform limit of sequences and series of functions and determine when the limit process preserves the regularity;
- generalize the convergence and topological properties acquired in the first part of the course from **R** to metric spaces and in particular to the space of continuous function on an interval;
- formulate an abstract mathematical argument in a rigorous way paying attention to the role of the hypotheses and to possible examples and counterexamples;

Course Content

This course treats the rigorous mathematical theory behind single-variable Calculus:

completeness of **R**, limits, continuity, differentiability, integrability, and the mutual relation between these concepts. The mathematical theory is presented in such a way that everything can later be generalised to the context of metric spaces. The space C^0[a,b] of real-valued continuous functions on an interval [a,b] will appear as the main example of such metric spaces for which the theory can be applied to solve differential equations via the Banach contraction principle.

Topics:

1. Suprema, infima and completeness of real numbers;
2. Limit of real sequences;
3. The Bolzano-Weierstrass Theorem and the Cauchy Criterion;
4. Applications of convergence to infinite series and to the Banach contraction principle;
5. Basic topology of **R**
6. Functional limits and (uniform) continuity;
7. Differentiable functions: linear approximation and the Mean Value Theorems;
8. The Riemann integral: construction, properties and the Fundamental Theorem of Calculus
9. Pointwise and uniform convergence of sequences of functions
10. Metric spaces: basic topology, completeness and the Banach contraction principle
11. Application to the space C^0[a,b] with the uniform distance: the Picard-Lindelöf Theorem on differential equations.

Additional Information Teaching Methods

Lectures, study sessions and tutorials (2+1+2 hours per week). You will hand in a homework assignment every other week. We expect you to dedicate in total about 10 hours per week to this course.

Method of Assessment

Your final grade is built up as follows:
A written midterm exam [40%];
A written final exam [50%];
Five written assignments [10%] (the best four count).

To pass the course your total score must be no less then 55%.

If you don't fulfil this requirement, then you can take the resit. In this case, your grade will be

A) either [10%] written assignments + [90%] resit
B) or [100%] resit, depending which of the two options results in a higher grade.

It is also possible to use the grade of the assignments of the past year. Write to the lecturer, if you want to use this option.

Entry Requirements

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Literature

For the majority of the course we study Stephen Abbott, Understanding Analysis, Springer, 2015, Second edition, ISBN 978-1-4939-5026-3.

For the last part on metric spaces, the content is based on excerpts from Chapter 1 and Chapter 5.1,5.3 of
Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and Sons, 1978, ISBN 0-471-50731-8.

Concise notes of the lecturer will be provided to keep track of the exact material discussed during the course.

Additional Information Target Audience

Bachelor Mathematics Year 1

Recommended background knowledge

The following background is necessary for this course:

1. Basic Concepts in Mathematics (or another course on general mathematical language, notation, and concepts, including proof by induction and elementary combinatorics).
2. Single Variable Calculus