MAT250 Calculus

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Introduction

Calculus is the rigorous mathematical study of *continuous change and motion*. Calculus is used extensively in mathematical modelling, and consequently it has wide applicability in many fields of business, engineering and science. For instance, maximising profit, minimising fuel consupmtion, calculating volumes of objects, etc., are routine applications of this streamlined mathematical theory. Surprisingly, much of this success depends on the careful usage of an elusive concept, the *infinitely small*. How infinity was tamed and put into good use is an exciting story, and those historical steps are retraced by students during every calculus course.

Objectives

This course aims to give students tools of mathematical modelling applicable to real-world problems and to enhance their mathematical thinking skills by transitioning them from the intuitive thinking to the precise definitions, theorems and proofs. It is primarily intended as a solid foundation in calculus. Both differential and integral calculus will be discussed, with emphasis on the former. In particular, the following – rather surprising – theorem connects the seemingly unrelated differential and integral calculus:

Theorem (Fundamental Theorem of Calculus)**.** If function f is continuous on the closed interval $[a, b]$, then

1. if $g(x) = \int_a^x f(t)dt$, then $g'(x) = f(x)$,
2. $\int_a^b f(x)dx = F(b) - F(a)$, where $F' = f$.

The course will provide a thorough explanation of this result describing the exact meaning of the obscure looking symbols above.

Learning Outcomes

On the successful completion of this course, it is expected that the students will be able to:

1. Demonstrate appropriate use of standard mathematical formulae and techniques in modelling real-world problems, and have insights into the process of building mathematical theories.

2. (a) Understand the meaning of the limit of a function.
   (b) Find limits of functions.
   (c) Understand the meaning of continuity.
   (d) Determine where a function is continuous.

3. (a) Understand the meaning of the derivative of a function.
   (b) Find the derivative of a function.

4. Apply correctly the techniques of differential calculus to problems involving optimization, curve sketching and rates of change.

5. Calculate basic integrals.

Tentative Schedule

Week	Lecture Topics
1	**Revision and roadmap:** quick review of functions, algebra, analytic geometry and trigonometry; big ideas of calculus.
2	**Functions and limits:** intuitive notion of the limit of a function; one-sided limits; vertical asymptotes; precise definition of the limit.
3	**Calculating limits:** limit laws; the squeeze theorem.
4	**Continuity:** continuity of functions; limits at infinity; horizontal asymptotes.
5	**Differentiation:** definition of the derivative; relationship between differentiability and continuity; derivatives of polynomials and exponential functions.
6	**Differentiation:** product and quotient rules; derivatives of trigonometric functions; chain rule; implicit differentiation; derivatives of the inverse trigonometric functions.
7	**Differentiation:** derivatives of logarithmic functions; logarithmic differentiation.
8	**MIDTERM TEST**
9	**Differentiation:** related rates; linear approximations and differentials; derivatives of hyperbolic functions and their inverses.
10	**Applications of differentiation:** maximum and minimum values; extreme value theorem; Rolle's theorem; mean value theorem; first and second derivative tests.
11	**Applications of differentiation:** indeterminate forms and L'Hospital's rule; curve sketching.
12	**Applications of differentiation:** optimization problems; Newton's method. Taylor and Maclaurin polynomials.

13	**Integration:** antiderivatives; indefinite integrals; definite integrals; relationship between indefinite and definite integrals; the Fundamental Theorem of Calculus.
14	**Revision**
15	**FINAL EXAM**

Textbook

James Stewart, Single Variable Essential Calculus, 2^{nd} International Edition (Metric Version) 2013, Thomson Brooks/Cole (ISBN-13: 978-1-133-526862-3)

Software

For demonstration purposes GeoGebra will be used. It is a multi-platform, open-source, freely available mathematics software package https://www.geogebra.org/. It is also recommended for students to experiment with functions and their derivatives, checking homework problems using the software package.

Assessment Components

Homework 30%; purpose: to encourage continuous learning throughout the semester;

Midterm exam 30%; purpose: to encourage and assess continuous learning throughout the semester and to provide feedback on the learning progress to lecturers and students;

Final exam 40%; purpose: to assess the extent to which learning outcomes of the unit have been achieved;

Exams are closed book, a non-programmable calculator without graphical display is permitted; derivation tables provided in the final exam.

Delivery format

The course will be delivered as a lecture, but questions are encouraged at all times. Some classes will have problem solving sessions.

Prerequisites

MAT150 College Algebra, or an equivalent precalculus course.