Mathematics for the Digital World

Level: 200

Credits: 3

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Introduction

Nowadays it is natural to order a product by tapping on the touchscreen of a smartphone. One can confidently enter bank details since the communication channel is encrypted and thanks to the efficient logistics of the delivery company the parcel arrives at the doorstep next day. How can this be done? If we look at all these technologies, from the data processing capabilities of computers and mobile communication to industrial optimization, we can always find some mathematical results as their core ideas. The actual details of course can be extremely complicated, however the underlying ideas are quite simple. The aim of the course is to show how the abstract mathematical knowledge is used in the real world by giving a hands-on experience through a selection of mathematical problems. These will be presented with assuming minimal background.

Description

The course consists of two parts: a low-level description of how computers work and a high-level explanations of some of the most important algorithms used in everyday life. The first part describes how information is represented, transferred and processed by computers. The second part is centered around a single mathematical object called *matrix*, a table of numbers. Matrices appear in many branches of mathematics and in efficient algorithms.

Learning Outcomes

By completing the course, the students will have extended mathematical knowledge and a clear understanding of the inner workings of today's information processing technologies. These skills

are becoming crucial to face the challenges of advanced technical civilization. In particular, it is expected that the students will be able to:

1. Manipulate mathematical representations of information:
   (a) Represent numbers with different base, perform binary arithmetic operations, encode simple images as sequences of numbers.
   (b) Construct and minimize basic logical circuits.
   (c) Construct optimal and error-correcting codes for transmitting information.
   (d) Describe and manipulate basic structures in number theory, and apply it to real world problems (e.g. cryptography).
2. Carry out operations on matrices and apply them to a variety of applications, namely:
   (a) Solve optimization problems by linear programming methods.
   (b) Understand basic concepts of graph theory and their applications, decide simple graph isomorphism problems.
   (c) Work with probability transition matrices.
   (d) Analyze simple games and find winning strategies.

Tentative Schedule

Week	Lecture Topics
1	**Introduction:** Abstraction and problem solving. Brief history of computing machines. Basic mathematical notation (sets, numbers, algebraic operations).
2	**Number systems and data representations:** Binary arithmetic, bitmaps.
3	**Logic and computing:** Mathematical logic, Boolean algebra.
4	**Digital circuits:** logic gates and Boolean functions; arithmetic by logical circuits; realizing digital computation in the Game of Life 2-dimensional world.
5	**Data transmission and compression** Optimal codings; error-correction; loss-less compression exploiting statistical redundancy; lossy compression of sound and images.
6	**Number Theory Basics:** prime numbers, modular arithmetic.
7	**Cryptography:** substitution ciphers; public key encryption, cryptocurrencies.
8	**MIDTERM TEST** covering weeks 2–7
9	**Matrix algebra** Multiplication and inverses. Linear transformations, systems of linear equations.
10	**Game theory** Mathematical models of conflict and cooperation. Zero-sum games.
11	**Linear Programming:** standard form, simplex algorithm.

12	**Graphs and Trees:** Graphs and adjacency matrices; social networks; search trees.
13	**Markov Chains:** Discrete time random processes; applications; Google's PageRank algorithm.
14	**Summary and revision** Additional problem solving session.
15	**FINAL EXAM** covering weeks 9–14

Textbook

Electronic lecture notes will be provided.

Software

GeoGebra is a free open-source mathematics software system. It will be used for demonstration, checking hand calculations and for some homework problems. The software package is web based, therefore no installation is necessary. www.geogebra.org
Golly is an open source, cross-platform application for exploring Conway's Game of Life and other cellular automata. golly.sf.net

Assessment Components

The assessment is based on problem solving.

Homework 30%; purpose: to encourage continuous learning throughout the semester and provide feedback;

Midterm and final exams 35% each; purpose: to assess the extent to which learning outcomes of the unit have been achieved.

Prerequisites

MAT150 College Algebra, or an equivalent course.